

Stochastic Reaction Network

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1 Introduction

The objective of this practical project is to program and study the variability of simple stochastic reactions, using the Gillespie algorithm. This practical work is based on [1]

```
function GILLESPIE( $\mathbf{x}_0$ ,  $\mathbf{k}$ ,  $T$ ,  $V$ )  
   $\mathbf{x} \leftarrow \mathbf{x}_0$   
   $t \leftarrow 0$   
  repeat  
    //Compute the reaction rates  
     $\{\lambda_{r_1}, \dots, \lambda_{r_J}\} \leftarrow \lambda(\mathbf{x}(t), \mathbf{k})$   
     $R_{tot} \leftarrow \sum_{r_i} \lambda_{r_i}$   
    //Draw next reaction time  
     $\delta_t \sim Exponential(\lambda = R_{tot})$   
    Draw a reaction  $r_j$  with Prob. prop. to  $\lambda_j$ .  
     $\mathbf{x} \leftarrow \mathbf{x} + V_{.,r_j}$   
     $t \leftarrow t + \delta t$   
  until  $t \geq T$   
end function
```

Fig. 1. Gillespie pseudocode

You are free to program these simple examples in your favourite language, but I'll only debug code written in my favourite language :)

2 Simple bio-chemical reactions

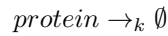
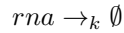
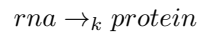
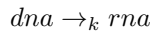
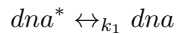
Write the programs to simulate the following bio-chemical reactions:

- $\emptyset \xrightarrow{k_1} A \xrightarrow{k_2} \emptyset$
- $A + B \xrightarrow{k} C$
- $A \xrightarrow{k} A + B$
- $A + B \xleftrightarrow{k_1} C \xrightarrow{k_2} D + E$

Execute your programs many times, and for each reaction compute the mean number of molecules and their standard deviations. Compare to the ODE results (you can write a simple Euler solver and/or solve the ODE analytically). Change the rate constants and analyze what happens.

3 Gene expression

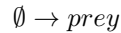
Let us study a simple stochastic gene expression system. DNA randomly alternate between two states: open and closed. Open DNA is transcribed into RNA, RNA is translated into a protein. Finally RNA and proteins are degraded. The system of reactions is detailed hereafter:



Keep k constant and record the amount of proteins along time for different rates k_1 . Repeat the experiment many times, just as if you were measuring the amount of molecules in different clonal cells. Compute the means and standard deviations. What do you see?

4 Lotka-Volterra

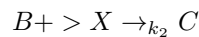
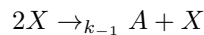
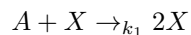
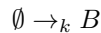
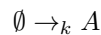
Simulate the Prey-Predator Lotka Volterra system:



Did you notice any difference with respect to the ODE solutions?

5 Bifurcation

Consider the following system:



Calculate the steady states of the system. Depending on the sign of $k_1 A^* - k_2 B^*$, the system has two stable steady states. Simulate different with different values for k_1 and k_2 . What do you see?

6 SIR system

Write the equations of the SIR system and simulate it using the Gillespie algorithm and the tau-leaping algorithm

References

1. Soula, H.: Stochastic chemical reactions. BioComp-5BIM-TD (2013)